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Comparison of correlations and experiment in opposing flow, mixed convection heat transfer in a vertical tube with Grashof number variation

DONALD D. JOYE

Department of Chemical Engineering, Villanova University, Villanova, PA 19085-1681, U.S.A.

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Abstract—Mixed convection heat transfer in a vertical tube with opposing flow (downflow heating) was studied experimentally for Reynolds numbers ranging from about 700 to 25 000 at constant Grashof number under constant wall temperature (CWT) conditions. The Grashof number was varied independently by adjusting steam pressure in the jacket of a vertical, double-pipe heat exchanger. In the turbulent region at $Re > 10\,000$ and in the asymptotic region at $Re < 4000$, the data agreed well with published correlations that omit buoyancy contribution. The 'mixed' convection region existed at Re between 4000 and 10 000. As the Grashof number was reduced in this Re range, the Nusselt number was also reduced. The Jackson and Fewster correlation and the Swanson and Catton correlation developed for opposing mixed convection flows in vertical conduits both predicted the data quite well, except near and into the asymptote region, for which these equations were not designed.

INTRODUCTION

Heat transfer situations in which natural convection and forced convection heat transfer mechanisms interact are termed 'mixed' convection. The literature in this field is extensive, because each case of geometrical variation must be treated separately. The focus of this work is on downward internal flow in a heated, vertical tube. Downflow heating is termed 'opposing' flow, because the natural convection currents are in the opposite direction from the forced flow. In contrast, upflow heating is often termed 'aiding' flow, because the natural convection currents are in the same direction as the forced flow. Heat transfer results for upflow heating are typically quite different than those for downflow heating, hence these comprise two separate cases. This investigation also included upflow heating experiments, but the upflow results and a comparison of the two are reported in a companion paper [1]. Vertical internal flow heat transfer has application to vertical shell-and-tube heat exchangers, nuclear reactors and some aspects of electronic cooling. An extensive review paper by Jackson *et al.* [2] summarizes most of the work in this field.

Research in mixed convection flows has been driven by the great complexity of the interactions between forced and natural convection, and in some cases the importance of the practical applications, such as nuclear power technology [3, 4] and chemical process technology where simultaneous heat and mass transfer occur. Comprehensive experimental data are still scarce. Recently, some investigators have recognized that significant heat transfer enhancement may be realized in vertical flow situations, and this is driving research on vertical, unbaffled shell-and-tube heat exchangers.

Correlations that predict the heat transfer characteristics in opposing flow in vertical conduits do exist. The Jackson and Fewster correlation is discussed in some detail by Jackson *et al.* [2], and another correlation based on surface renewal theory has been introduced by Swanson and Catton [5]. Although the Jackson and Fewster correlation has been based on a fair amount of data, and the Swanson and Catton correlation is somewhat similar, few independent tests of these relationships have been reported.

Joye *et al.* [6] present data from an experimental study of heat transfer characteristics for upflow and downflow heating in a vertical tube under constant wall temperature boundary conditions and high Grashof number. Three regions are shown to characterize the heat transfer in this situation. The turbulent flow region occurs at Reynolds numbers greater than about 10 000, where the Nusselt numbers for vertical mixed convection heat transfer agree with those predicted from well known forced flow correlations, e.g. the Sieder–Tate or any of the newer relationships [7]. Mixed convection is not important in this turbulent region, because the hydrodynamic turbulence at these Reynolds numbers is much stronger than the natural convection mechanism and dominates the heat transfer. At low Reynolds numbers, the asymptotic region exists and represents heat transfer and flow conditions where the outlet temperature approaches the wall temperature. This region was anticipated by both Martinelli *et al.* [8] and McAdams [9], where it is discussed in some detail. At Reynolds numbers between the

NOMENCLATURE

C_p	fluid heat capacity [kJ kg^{-1} K^{-1}]
D	tube diameter [m]
g	gravitational acceleration [m s^{-2}]
Gr_{fD}	Grashof number based on fluid properties evaluated at the average film temperature and tube diameter, $\rho^2 g D^3 \beta \Delta T/\mu^2$
Gz	Graetz number, wC_p/kL
h	film heat transfer coefficient [W m^{-2} K^{-1}]
k	fluid thermal conductivity [W m^{-1} K^{-1}]
L	heated length of tube [m]
Nu	Nusselt number, hD/k
Pr	Prandtl number, $\mu C_p/k$
Re	Reynolds number, $Dv\rho/\mu$
T	temperature [K]
ΔT	temperature difference [K] average wall to average bulk fluid for Grashof number, unless otherwise noted
v	average fluid velocity [m s^{-1}]
w	mass flow rate [kg s^{-1}].

Greek symbols

β	volume expansivity [1/K]
μ	viscosity [Pa s^{-1}]
ρ	density [kg m^{-3}].

turbulent and the asymptotic regions, the 'laminar' mixed convection region exists, where both natural convection and forced convection mechanisms are the same order of magnitude and interact in complex ways. The authors show that the parameter Gr/Re^2 which differentiates the region of mixed convection from purely forced convection is a range of values around 1.0, i.e. that a value of 1.0 for this parameter is not a limit for mixed convection flows, as the older literature suggests.

The transitions between the three regions are affected by several factors. Marcucci and Joye [10] showed these transitions are L/D dependent when length-based Grashof number is used, but using a diameter-based Grashof number may make this dependency rather weak. The transition points must also depend on Grashof number. In [10] the experiments were done at a single Grashof number, so this dependency has yet to be determined. Although both correlations of Jackson and Fewster, and Swanson and Catton are based on experimental data, that of Swanson and Catton is based on a particularly narrow range of Reynolds numbers. Thus, one object of the present work is to further develop a data base that includes Grashof number as a parameter as well as Reynolds number with as wide a variation as possible, in order to explore more completely the significance of these parameters.

THEORETICAL BACKGROUND

Two correlations exist for downflow, vertical mixed convection in a tube. The correlation of Jackson and Fewster [2] is given below.

$$Nu/Nu\,(\text{forced}) = (1+4500\,Gr_{\text{barD}}/Re^{2.625}Pr^{0.5})_b^{0.31} \quad (1)$$

where subscript "b" refers to properties evaluated at the bulk average temperature, and Gr_{barD} is defined by the integrated density term:

$$Gr_{\text{barD}} = \rho(\rho-\rho_{\text{bar}})gD^3/\mu^2 \quad (2)$$

and

$$\rho_{\text{bar}} = \left(\int_{T_w}^{T_b} \rho\, dT\right) \Big/ (T_w - T_b) \quad (3)$$

and the forced flow model is the Petuckov–Kirillov model given by the following:

$$Nu\,(\text{forced}) = \frac{Re\,Pr\,C_f/2}{12.7(C_f/2)^{1/2}(Pr^{2/3}-1)+1.07} \quad (4)$$

where

$$C_f = 1/(3.64\log_{10} Re - 3.28)^2. \quad (5)$$

A similar equation developed by Swanson and Catton [5] for flow in vertical channels of rectangular cross-section is shown below. This has an additional term and slightly different Prandtl number dependence. The Reynolds number formulation originally contained the well known hydraulic diameter, thus this equation can be applied to tubes as well.

$$Nu = 0.0115Re^{0.8}Pr^{0.5}\{1+[1-(696/Re^{0.8}) + 8300Gr/Re^{2.6}(Pr^{0.5}+1)]^{0.39}\}. \quad (6)$$

These correlations use as base case the turbulent forced-flow only correlation from which increases in heat transfer due to natural convection effects can be followed. In this work we prefer the Sieder–Tate equation with viscosity correction as the base case,

$$Nu/Pr^{1/3}\phi_v^{0.14} = 0.023Re^{0.8} \quad (7)$$

which is often referred to shorthandedly as the "0.023" equation. For heating situations the viscosity correction factor is often folded into a slightly different power for Pr. The Swanson and Catton base case shows this preference; Jackson and Fewster choose the Petuckov–Kirillov relationship, which differs only slightly from the Sieder–Tate. Thus, the base cases are all comparable.

Martinelli *et al.* [8] and McAdams [9] both show an asymptotic relationship for constant wall temperature (CWT) conditions at low Reynolds numbers. This is a result of losing ΔT driving force at the exit, and consequently would not be expected to hold for uniform heat flux (UHF) experiments, which maintains ΔT driving force. The equation takes various forms, but fundamentally it is

$$Nu_{\text{asymptote}} = (2/\pi)\, Gz \tag{8}$$

where Gz is the well known Graetz number. When Re is used as an independent parameter as we prefer, the asymptote is Pr number dependent,

$$Nu_{\text{asymptote}} = 0.5RePrD/L \tag{9}$$

which translates to

$$Nu_{\text{asymptote}}/Pr^{1/3}\phi_v^{0.14} = 0.5RePr^{2/3}D/L \tag{10}$$

in the equivalent Sieder–Tate style formulation.

EXPERIMENTAL METHOD

The apparatus employed for the present study was identical to that of Joye *et al.* [6], except that the steam condensing in the annulus of a copper–copper, double-pipe heat exchanger was routed to a Nash vacuum pump, and a control system was installed on the steam line to control steam pressure and its distribution in the jacket. Figure 1 shows the system and features. A vacuum gauge measured the (vacuum) pressure in the jacket, and a pressure gauge measured

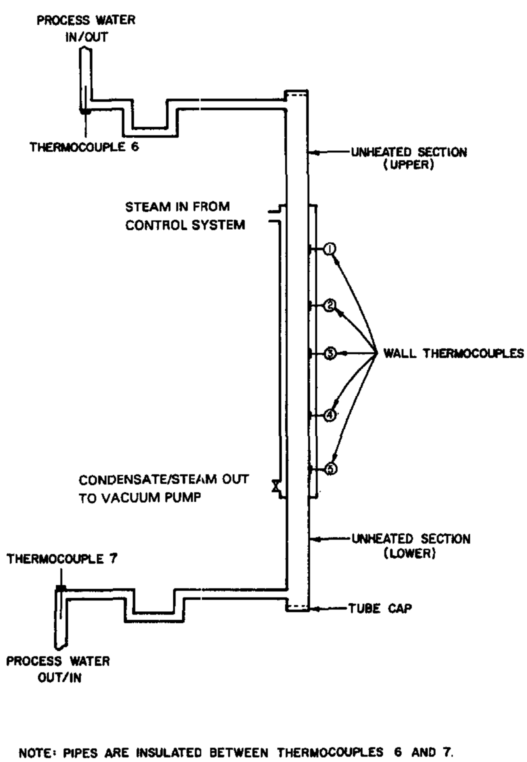

Fig. 1. Experimental apparatus. Test section detail.

positive steam pressure when that was used. Since five thermocouples were used to measure wall temperature, the vacuum pressure itself had little significance other than being constant for a given run. The vacuum pressures in the jacket were 99.6, 67.4, 33.6 and 30.2 kPa absolute pressure (0.5, 10, 20 and 21 in Hg vacuum, respectively). A run at 205 kPa (15 psig) steam pressure was included to extend the range of the present data. Any particular steam pressure value was easily held constant by the control system. Very probably steam flowed through the jacket as well as condensed in it, but this does not affect the results since the wall temperatures were measured directly. Constant wall temperature could be maintained and checked as well as in the previous investigation [6]. At low pressures the steam can often condense at the inlet only, and when this happened, the steam valves were adjusted to provide sufficient flow so that constant wall temperature was evident by readings on the digital thermometer. The lowest pressures were at the limit of the apparatus capability. Wall temperatures and hence Grashof numbers could be changed easily by adjusting the steam pressure within the limits of the equipment.

Copper–constantan theromocouples were used as before to measure both wall temperature and inlet–outlet temperatures. The unit was insulated to minimize heat loss, particularly important at the outlet. Mixing elbows were used to get a well-mixed outlet temperature in both upflow and downflow. Two rotameters were used to measure the flow rates at high throughput and at low throughput. The L/D of the heated section was 49.6; the inside diameter of the central tube was 0.032 m (1.265 in.). Water in the tube was prevented from boiling or degassing by pump pressures of 150–200 kPa gauge (20–30 psig) as previously [6].

Heat transfer data including all dimensionless groups, coefficients, etc. were calculated using spreadsheet software. Heat transfer coefficients (film coefficients) were calculated from Newton's Law of Cooling, where the heat rate (W or Btu h^{-1}) was calculated from the temperature rise and the flow rate of water, and an arithmetic average temperature driving force was used. (The log–mean temperature driving force was also used for comparison.) The Nusselt number was based on the arithmetic average (bulk fluid) temperature, and the Grashof number was based on properties evaluated at the film temperature, defined as the arithmetic mean of average wall temperature and bulk average fluid temperature, thus it is an average Grashof number. This was chosen to best represent the actual temperature of the fluid in the film region close to the wall where the buoyancy effects exist. Others have used the wall temperature or the bulk average fluid temperature, both of which have their advantages. Jackson *et al.* [2] prefer a Grashof number defined by average density in the film, but do not do the same for viscosity. The Grashof numbers of the present work could be converted to other bases, since all the appropriate temperatures were known.

RESULTS AND DISCUSSION

Water was used as the test fluid, and the Prandtl number (an uncontrolled parameter) varied from about 2 at low Reynolds numbers (hottest fluid) to about 8 at high Reynolds numbers depending upon the wall temperature. The corresponding viscosity ratio, ϕ_v = the viscosity at the bulk average temperature divided by that at the wall temperature, varied from about 1.2 to about 2.5. The Grashof numbers based on diameter and fluid properties at the average film temperature were constant for each run to within about $\pm 20\%$. Grashof numbers at Reynolds numbers greater than about 12 000 differed from the average for that run by greater than 20%, but this does not matter since the effect of *Gr* in this range (turbulent flow) is negligible. Table 1 gives the average Grashof number for each run at different temperature bases as a function of shell-side steam pressure. The corresponding Grashof number based on length can be computed easily, since $L/D = 49.6$ for all cases. With reference to the table, the Grashof numbers calculated by the Jackson and Fewster method are about 60% of the *Gr* based on fluid properties evaluated at the bulk average temperature and almost an order of magnitude less than *Gr* based on fluid properties taken at the film temperature.

Heat transfer results for downflow heating

A composite of the downflow heating heat transfer results is shown in Fig. 2, where the quantity $Nu(\text{ave})/Pr^{1/3}\phi_v^{0.14}$ is plotted against Reynolds number with Grashof number as a parameter. *Nu*(ave) was calculated on the basis of average heat transfer coefficient for the whole channel. The Grashof number (also an average parameter) changes as a function of steam pressure in the shell. Each set of data was taken with essentially constant Grashof number, except for the fully turbulent regime where buoyancy effects are negligible anyway. Although Grashof numbers decrease an order of magnitude, the corresponding decrease in Nusselt number is about half an order of magnitude. We were not able to get uniform heating of the tube at steam pressures less than about 30 kPa absolute, and the lowest approach temperature we were able to achieve was about 40°C. At this level the Grashof number is still rather high ($Gr_{fD} = 1 \times 10^7$), and natural convection effects are still quite prominent, giving Nusselt numbers two to six times higher than the laminar forced convection correlation. Thus, forced convection correlations will be in significant error for vertical, internal flows with heat transfer if *Re* is less than about 12 000. For Reynolds numbers above 12 000 the "0.023" equation predicts a bit low for the data, but this is typical, and others have used slightly different coefficients, e.g. 0.027, to get a better fit.

Table 1. Grashof number as a function of (absolute) steam pressure in the shell. *Gr* calculated on film temperature basis (Gr_{fD}), bulk average temperature basis (Gr_{bD}) and Jackson's density basis (Gr_{barD})

Steam P(abs) [kPa]	Gr_{fD}	Gr_{bD}	Gr_{barD}
205	1.00×10^8	3.20×10^7	2.21×10^7
99.6	7.65×10^7	1.95×10^7	1.04×10^7
67.4	3.15×10^7	0.80×10^7	0.65×10^7
33.6	1.45×10^7	0.48×10^7	0.29×10^7
30.2	1.00×10^7	0.32×10^7	0.24×10^7

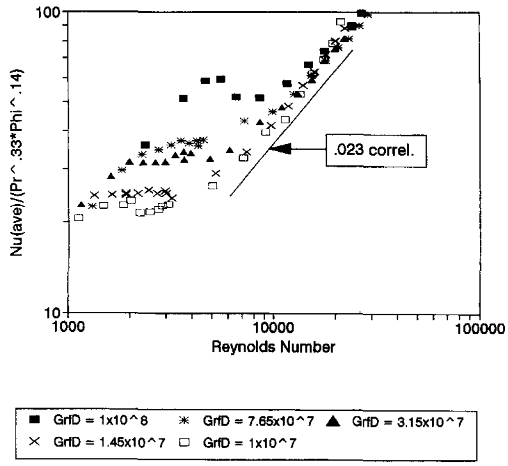

Fig. 2. Opposing flow (downflow heating) heat transfer results with different Grashof numbers resulting from different wall heating.

One notices that the curves seem to be shifted vertically only as *Gr* changes, and the slope with respect to *Re* is for the most part virtually flat in the mixed convection region. At the highest *Gr* condition, however, the slope with respect to *Re* is negative, which is consistent with previous literature.

One can see how the data approach both limiting regions, the turbulent region at high Reynolds numbers and the asymptote region at low Reynolds numbers. In the low Reynolds number region high temperatures exist in all cases, and the *Pr* differences between the four runs is small; *Pr* ranged from 2 to 3.5. Thus, the asymptote lines would be very close together, and the data appear to collapse to a single line.

Every experiment is subject to experimental error. The major sources of error in these experiments were time to steady state (which varies widely with flow rate) and flow rate variation or drift during a particular data collection interval. Compared with these, errors in temperature measurement (± 0.2°C) have only a small effect on the data. Gross errors were eliminated by checking the smoothness of a plot of outlet temperature vs flow rate. Outlier points were repeated with exacting flow control. Time to steady

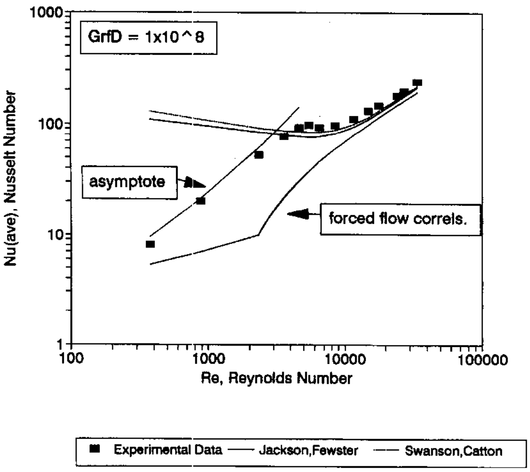

Fig. 3. Downflow heating data compared with correlations of Jackson and Fewster, and Swanson and Catton. Shell-side steam pressure = 205 kPa ($Gr_{fD} = 1 \times 10^8$).

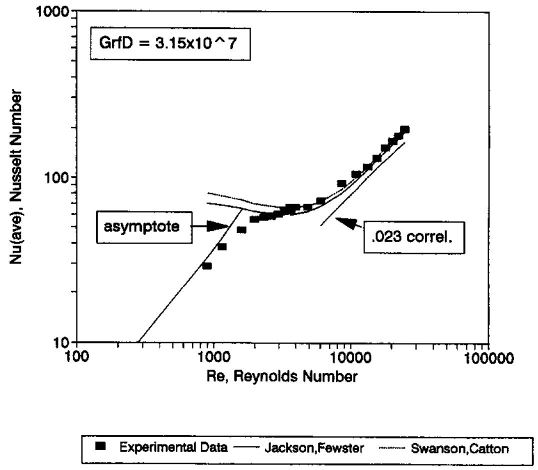

Fig. 5. Downflow heating data compared with correlations of Jackson and Fewster, and Swanson and Catton. Shell-side steam pressure = 67.4 kPa ($Gr_{fD} = 3.15 \times 10^7$).

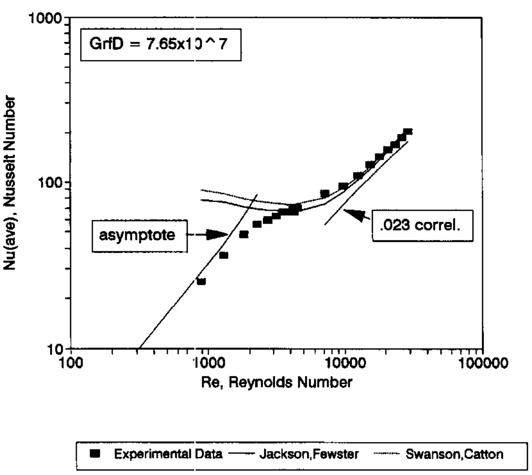

Fig. 4. Downflow heating data compared with correlations of Jackson and Fewster, and Swanson and Catton. Shell-side steam pressure = 99.6 kPa ($Gr_{fD} = 7.65 \times 10^7$).

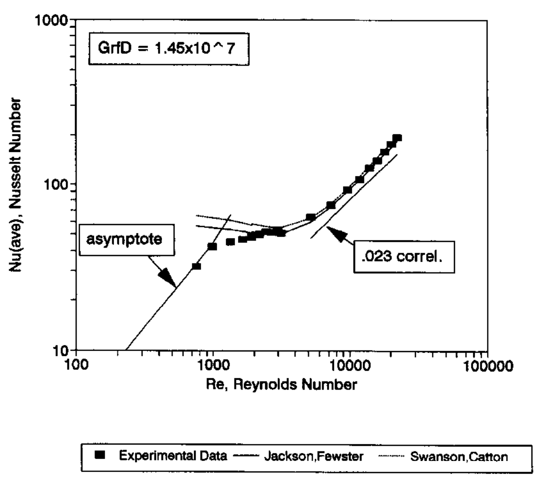

Fig. 6. Downflow heating data compared with correlations of Jackson and Fewster, and Swanson and Catton. Shell-side steam pressure = 33.6 kPa ($Gr_{fD} = 1.45 \times 10^7$).

state was almost instantaneous for the high Reynolds number data, but about 3 min were needed to achieve steady state at the lowest Reynolds numbers.

Downflow correlation comparison

Figures 3–7 show how the correlations of Jackson and Fewster and Swanson and Catton fit the downflow heating data of the present investigation. Both correlations seem to fit quite well. The Swanson and Catton model predicts about 7% higher than Jackson and Fewster. The Jackson and Fewster correlation seems to fit our data somewhat better, except for the high *Gr* case where Swanson and Catton predicts very closely and Jackson and Fewster is about 7% low, relative to the data. As the Grashof number changes both models follow the data quite well for both *Gr* and *Re* variation until the asymptote. Here both models fail to bend into the asymptote, as shown particularly in Fig 3. This is not surprising, because neither model

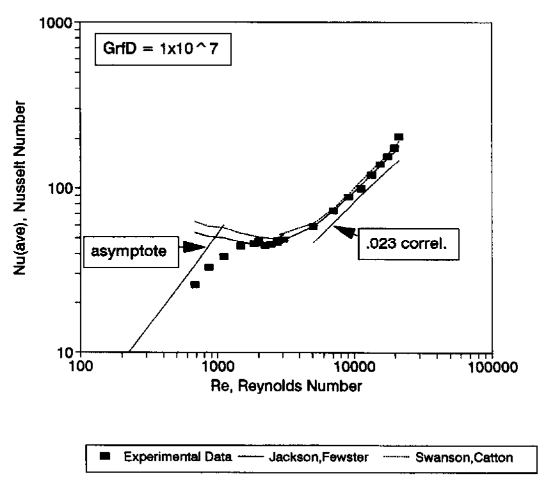

Fig. 7. Downflow heating data compared with correlations of Jackson and Fewster, and Swanson and Catton. Shell-side steam pressure = 30.2 kPa ($Gr_{fD} = 1.00 \times 10^7$).

takes the asymptote region into account. Therefore the range of applicability for the models extends only to the intersection with the asymptote.

As $Gr \to 0$ Swanson and Catton fails at $Re^{0.8} \leqslant 696$, because the $696/Re^{0.8}$ term in the brackets goes negative. As $Gr \to 0$ Jackson and Fewster reduces to the forced flow correlation. As $Re \to 0$, both models blow up (the last term in Swanson and Catton dominates). However, at $Re = 0$, Swanson and Catton reduces to the forced flow correlation. Although both models predict well in the range of mixed convection investigated here, each has its weaknesses. What is really needed is a model that bends into the asymptote at low Re and reduces to the turbulent forced flow correlation at high Re. With a bit more Gr parametric data and a little more thinking, this problem could soon be solved for the constant wall temperature (CWT) boundary condition.

CONCLUSIONS

(1) The data presented for opposing flow with Grashof number as a parameter show reduced heat transfer enhancement as Grashof number is lowered, as might be expected. The data quantify the effects for the given conditions.

(2) Tests of the theories of Jackson and Fewster, and Swanson and Catton show both theories seem to fit the data quite well with respect to Reynolds numbers from the asymptote region right into the forced convection turbulent region, but do not fit the asymptote region at all.

(3) Both theories seem to follow the data well for Gr_{fD} ranging from 1×10^7 to 1×10^8.

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